Macroscopic Quantum Coherence

R. A. Paju
Queens' College, Cambridge

Literature Review
Supervisor: M. J. W. Dodgson (TCM)

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Except where specific reference is made to the work of others, this work is original and has not been already submitted either wholly or in part to satisfy any degree requirement at this or any other University.

Risto A Paju

Abstract

The concept of macroscopic quantum coherence has been reviewed by an investigation into the theoretical work on the subject. After Schrödinger's philosophical pondering with the "cat paradox", the topic has been taken into more practical consideration in recent decades. The main problem has been how to distinguish between proper quantum superpositions and statistical mixtures. To analyze this, A. J. Leggett and others have developed new methods such as the concept of disconnectivity. Another important theme is the problem of decoherence, which is responsible for the practical difficulties in attaining macroscopic quantum superpositions.

In addition, the relevant laboratory work based on this theoretical construction has been examined. In those experiments, a superconducting loop interrupted by Josephson junctions is used to create a double potential well of a single macroscopic variable Φ, the magnetic flux trapped by the loop. The two bound states can form coherent superpositions by tunneling between the wells. An energy difference observed between the even and odd superpositions indicates that a statistical mixture is not an adequate model for the system.

Hence, the 'paradox' has finally been lifted. The superposition of macroscopic states has been shown to be a practically observable phenomenon: The two states in the superconducting loop differ by a current of order 1 μA, an essentially macroscopic quantity. Such loops will play a central role in the development of quantum computers, massively parallel calculating devices originally proposed by Feynman.

1 Introduction

Successful as quantum mechanics (QM) has been in explaining known physical phenomena, and predicting new ones to a spectacular precision, it has since its beginning been riddled with philosophical dilemmae. Probably the most famous discussion on the non-intuitiveness of QM was the paper by Schrödinger [1] where the infamous cat was introduced, and the rivalry between different interpretations of the wavefunction has yet to end.

One of the main concerns for people interpreting QM has been, whether there is something intrinsically absurd about a QM superposition of macroscopic states. For a long time the problem has been out of the reach of experimentalists because of decoherence effects. However, techniques have improved markedly in recent years, and it has finally become possible to test the question in practice [2, 3]. But despite the technological advances, a lot of effort has been put into choosing the right types of experiment. A significant part of this paper will be devoted to the theoretical constructions leading from the early philosophical problems to today's microfabricated devices in cryogenic laboratories.

The main question of the present paper is whether QM, in particular the linear Schrödinger equation, can be extrapolated to apply to macroscopic systems. For this is it crucial to define what is mean by "macroscopic" in the sense relevant to the question; simply a system being large and tangible is not sufficient, as will be seen.

2 Theoretical background

2.1 Interpretation of the wavefunction

In his 1935 paper [1] Schrödinger discussed in detail the meaning of the wavefunction ψ of a QM system. One of its disturbing features is its abrupt change when a measurement is made: it changes into an eigenstate of the operator corresponding to the measured quantity. However, Schrödinger emphasizes the basic postulate of QM that ψ describes the maximal knowledge any observer can possibly have of the system. What happens in the "more real" level is out of our reach, and changes in ψ simply infer a change in our knowledge.

The significance of the above to our present problem is that all philosophical worries can be left aside. The question is merely on the wavefunction. Superpositions like $(|0\rangle \pm |1\rangle)/\sqrt{2}$ where $|0\rangle$ and $|1\rangle$ are macroscopically distinct may seem strange from the wavefunction perspective, but the ultimate reality may be something simpler. Of course, this does not directly solve the "cat paradox" where "dead" and "alive" are not unique quantum states, and the situation is immensely more complex.

2.2 Meaning of "macroscopic"

The concept of "macroscopic" is not in general clearly agreed on among physicists. For the purpose of this subject - the extrapolation of the Schrödinger equation - a special definition has been proposed by Leggett [4]. To begin with, it is instructive to look at established phenomena which rely on QM and manifest themselves in macroscopic effects. One example is superconductivity in metals as described by the Bardeen-Cooper-Schieffer (BCS) theory [5]: Electrons in superconductors travel in pairs (the so-called Cooper pairs) which behave like bosons. In a slightly simplified form, a BCS wavefunction is a product of two-particle wavefunctions, one for each pair of electrons. Thus the Schrödinger equation can be treated by the separation of variables, which leads to separate Schrödinger equations for each Cooper pair. Therefore, even though the supercurrent is a macroscopically observable phenomenon (involving a macroscopic number of electrons) the relevant Schrödinger equations only apply in a microscopic level.

2.3 Coherence: Pure states and statistical mixtures

As is the case with optics, coherence effects are those where the phases of interacting wavefunctions have definite relations. A simple way of putting this, due to Albrecht [6], for wavefunctions of the type

$$|\psi\rangle = \sum_i c_i |i\rangle, \; \sum_i |c_i|^2 = 1 \tag{1}$$

is that coherence effects are such that depend on the probability amplitudes c_i rather than only the probabilities $|c_i|^2$.

When the basis vectors $|i\rangle$ of the so-called pure state (eq. 1) are the eigenstates of an operator corresponding to an observable A with eigenvalues a_i, standard QM predicts that a measurement of A yields the result a_i with the probability $|c_i|^2$. Hence it might be interpreted that the system is already in one of the states $|i\rangle$ with the named probability, and the act of measurement simply removes our uncertainty [4]. This is the case of a statistical or incoherent mixture of states, and our key problem is whether it can be distinguished from a coherent one. A difference is indeed seen in the density matrices of the two states:

$$\begin{array}{lcll} \rho_{ij} & = & c_i^* c_j & \text{(pure/coherent)} \\ \rho_{ij} & = & |c_i|^2 \delta_{ij} & \text{(stat.mixture)} \end{array} \tag{2}$$

Hence the two states must be physically distinguishable [7].

In another treatment by Leggett and Garg [8], it is shown by a method analogous to Bell inequalities [9] that in an idealized quantum coherent situation, the predictions of QM differ from those obtainable from macrorealistic models (i.e. statistical mixtures). Suitable experiments are also discussed.

2.4 Disconnectivity

Section 2.2 has been provided to exemplify the fact that not all macroscopically observable quantum phenomena require the extrapolation of the Schrödinger equation into the macroscopic realm. However, in order to find systems which could test the validity of a macroscopic Schrödinger equation, a more rigorous approach is necessary. This is essentially where Leggett [4] introduces the concept of disconnectivity.

This idea begins with a fairly standard QM theorem that, for a system of N particles, "no measurement of any correlation between less than N particles will distinguish the pure state from a mixture" [4], despite the physical difference implied by the density matrices (eq. 2).

Consider a system of N' identical bosons (to escape slight complications with fermions or distinguishable particles). For any $N \leq N'$ a "reduced entropy" is defined as

$$S_N \equiv -\mathrm{Tr}(\rho_N \ln \rho_N)$$

where ρ_N is the reduced density matrix in which only the N particles are considered. The new quantity

$$\delta_N \equiv \frac{S_N}{\min_{(M)}(S_M + S_{N-M})}$$

is defined, so that δ_N is unity when both the numerator and the denominator are zero, and $\delta_1 \equiv 0$; then the disconnectivity D is defined to be the largest integer N for which δ_N is smaller than some given fraction a.

For simple system it is easy to show [4] that D is large when there are many-particle correlations quantum rather than statistical in nature. Furthermore, macroscopic quantum coherence (MQC) effects require D to be a macroscopic number [4].

2.5 Decoherence

In an abstract level, the most promising system to exhibit MQC would be a double potential well of one macroscopic degree of freedom q [4]. Tunneling between the two bound states would, in principle, allow for coherent superpositions of the two states. However, the practical reason why, even more generally, macroscopic quantum phenomena have so far not been observed, is the phenomenon called decoherence. It takes place in systems that are coupled to their environment by a macroscopic number of degrees of freedom. Each coupling effectively represents a QM measurement whereby the original wavefunction "collapses" to an eigenstate of the relevant operator. When coherent states are created, they will quickly decohere into statistical mixtures.

Moreover, the same dissipation that is responsible for decoherence, also reduces the tunneling probability [10]: For a linear dissipation coefficient η, the tunneling rate is multiplied by $\exp\{-A\eta(\Delta q)^2/\hbar\}$ where Δq is the tunneling distance and A a factor of order unity. In most practical situations, these two effects ensure that the system is found in a bound state in either well.

3 Experiments

In recent years, the double-well system of the kind mentioned above has been realized in practice in the form of a superconducting loop with one or more Josephson junctions. As analyzed by Tian et al. [11], it has been possible to attain a sufficiently low level of decoherence by a microfabricated solid state loop with dimensions of order 1 μm.

3.1 Principle

In a plain superconducting loop, the magnetic flux Φ through the loop is quantized [5]: As the electron pair wavefunction must be single-valued, its phase can only change in integral multiples of 2π over a closed path. Hence $\oint_{\mathrm{loop}} \mathbf{j} \cdot d\mathbf{l}$ where $\mathbf{j}$ is the particle flux, is quantized and it follows via Stokes' theorem that the resulting magnetic flux Φ is a multiple of $\Phi_0 \equiv h/2e$.

The "external flux" Φ_x is defined as the product of the applied magnetic field and the loop area. In other words, it is the flux that would be there if the loop were removed. Because Φ_x can have a continuous range of values, it is not generally equal to Φ. The result is a multitude of crossing energy levels as shown in fig. 1. Each parabola represents a unique bound state in a potential minimum, with a fixed supercurrent.

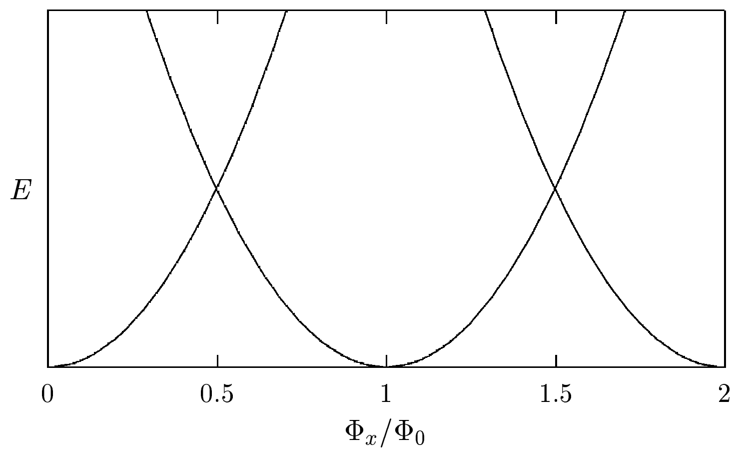

Figure 1: Energy levels of a plain superconducting loop in an external magnetic flux Φ_x where Φ_0 is the flux quantum.

The introduction of a Josephson junction enables tunneling between these bound states. As shown by Josephson [12], the Cooper pairs of electrons can tunnel through an insulating layer called the Josephson junction, even without a potential difference, provided there is a change in the phase of the Cooper pair wavefunction. In the energy level diagram, the tunneling is manifested in an anticrossing of levels (fig. 2) [13]. The double well we have been looking for, has the flux Φ through the loop as its main degree of freedom. That Φ need not be a multiple of Φ_0 is essentially due to the additional phase change over the Josephson junction. The potential energy is depicted in fig. 3 and given by

$$U = U_0 \left\{ \frac{1}{2} \left(\frac{2\pi(\Phi - \Phi_x)}{\Phi_0} \right)^2 - \beta_L \cos \frac{2\pi\Phi}{\Phi_0} \right\}$$

where $U_0 \equiv \Phi_0^2/4\pi^2 L$ and $\beta_L \equiv 2\pi L I_C/\Phi_0$, L being the inductance of the loop and I_C the Josephson critical current of the junction [13]. The squared part is the energy for a plain superconducting loop (as described in fig. 1 where Φ is an integral multiple of Φ_0) and the cosine term is due to the Josephson junction. This makes it more obvious that the Josephson junction is necessary to enable tunneling.

The bound states $|0\rangle$ and $|1\rangle$ correspond to macroscopically distinct supercurrents, possibly flowing in opposite directions. When the dissipation (coupling to environment) is weak, a bound state can tunnel into the other well with little energy loss. Consequently, the state can tunnel back and forth between the wells. In the limit of

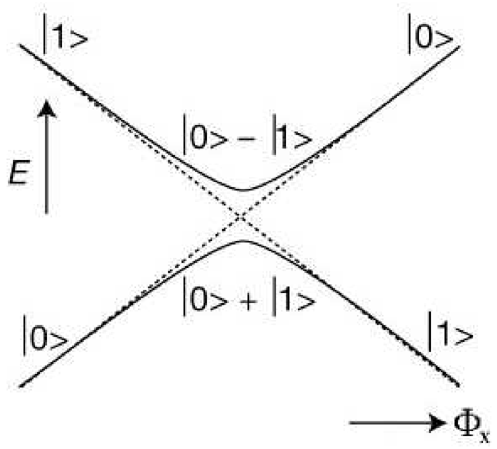

Figure 2: Anticrossing of energy levels. $|0\rangle$ and $|1\rangle$ are bound states of different currents. Dashed lines indicate 'classical' levels with no tunneling. (Adapted from [13])

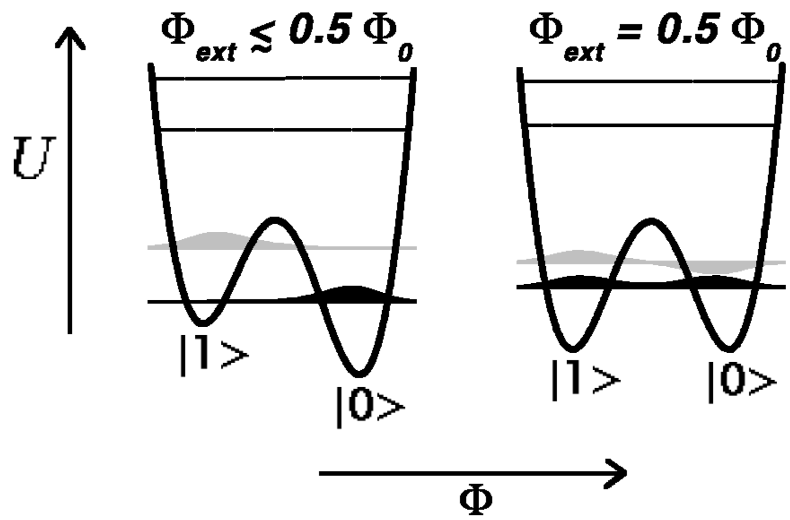

Figure 3: Double potential well of the flux Φ through the superconducting loop with a Josephson junction. (adapted from [15])

very weak dissipation, the phenomenon is called resonant tunneling which results in superpositions of the form $(|0\rangle \pm |1\rangle)/\sqrt{2}$. The odd combination has a higher energy, which is partly due to the higher kinetic energy when there is a node in the wavefunction. This brings us back to the meaning of coherence (section 2.3): in a statistical mixture with no definite phase relationship between $|0\rangle$ and $|1\rangle$, the odd and even combinations would be indistinguishable. But if an energy difference is observed, it can be concluded that the state is in fact a coherent quantum superposition.

3.2 Practical realizations

The above arrangement of microfabricated superconducting loops has been used by both Friedman et al. at the State University of New York, Stony Brook, NY [13] and Mooij et al. at the Delft Institute, The Netherlands [14, 15, 16, 17] to observe the desired anticrossing of energy levels. The loop has been coupled to a SQUID[1] for the measurement of Φ_x; this obviously has an effect of increased decoherence, but it is sufficiently small not to disturb the basic operation.

The loop is first prepared in one of the bound states, by keeping the external magnetic field on while the loop is cooled to a superconducting mode. Then the system is irradiated with microwaves of a range of frequencies; again, the energy of these is kept low in order to minimize decoherence. From the intensity of transmitted waves, an absorption spectrum is constructed. The energies of transitions between the even and odd states are observed as absorption peaks, while Φ_x has been measured by the SQUID. Varying Φ_x, graphs of the type in fig. 2 are constructed, from which the presence or absence of anticrossing can be deduced.

The main difference between the experiments of the Stony Brook and Delft groups is in the precise form of the double well and its energy levels. The Delft group tuned the system to have Φ very close to $\frac{1}{2}\Phi_0$ in order to make the potential symmetric (fig. 3). Then there could be coherent oscillations between the ground states in each well. On the other hand, the Stony Brook group used excited states in the wells: then a matching pair of energy levels could be found even for a significantly non-symmetric potential (fig. 4).

Both groups have observed the anticrossing of the energy levels, which is substantial support for a quantum coherent model. In addition, both groups report that the states $|0\rangle$ and $|1\rangle$ have a difference in the circulating current in the order of microamperes, a macroscopically measurable value.

4 Discussion

4.1 General remarks

The experimental evidence clearly speaks for the notion that coherent quantum superpositions are not limited to the atomic scale. While the picture from these observations may not be complete – i.e. perhaps there is something more complicated than a simple superposition – the macrorealistic, statistical interpretation is definitely ruled out.

[1] A superconducting loop with two Josephson junctions used for very precise measurements of magnetic flux. See e.g. Feynman [5] for details (although he does not use the name SQUID).

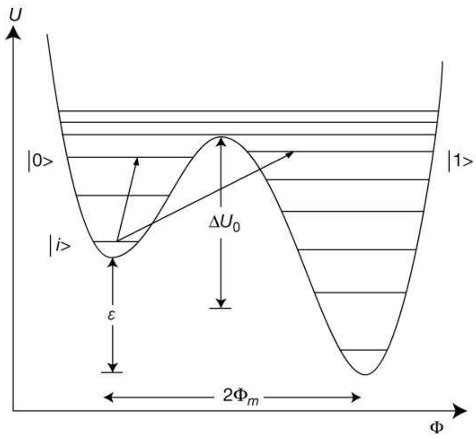

Figure 4: The double potential well by the Stony Brook group. (adapted from [13])

It is interesting to note that previously there has been substantial debate whether MQC effects are possible even in principle. The debate has mostly concerned philosophical considerations and 'paradoxes' such as Schrödinger's cat [1]. However, a proper QM analysis in the wavefunction level has shown no sound implications against MQC [2, 3].

Whether the above discussed effects are truly macroscopic may remain debatable. The Stony Brook group reports that the difference measured between the two quantum states corresponds to a current of 2...3 μA with a collective motion of about 10^9 Cooper pairs, while the total magnetic moment of the loop changes by about $10^{10}\mu_\mathrm{B}$. However, the usual requirement for "macroscopic" is a number of particles of the order 10^{23}. Of course this is only relevant in the human scale, as there cannot be a universal definition. Nevertheless, the question remains whether QM can be extrapolated into arbitrarily large values of the quantities involved.

4.2 Potential application: Quantum computing

The idea of quantum computation was first proposed by Feynman [18] and has since been subject to considerable research, particularly in recent years. While the bits in a conventional digital computer have strictly either of two allowed values (0 and 1), quantum bits or qubits have superpositions of the two. In principle, this allows multiple calculations to be carried out in parallel. For example, a two-qubit system can be in a superposition of the four states $|00\rangle$, $|11\rangle$, $|10\rangle$ and $|01\rangle$, hence being capable of four operations in parallel. By similar reasoning, an N-bit computer has a parallelism of 2^N. As has been pointed out e.g. by Albrecht [6], a quantum computer can perform no calculations that cannot also be done on a conventional computer (since one can, in principle, model a full QM environment on the latter). The main advantage of a quantum computer is that in certain kinds of calculation it can be enormously faster.

Incidentally, the experimental efforts of the Delft group are primarily directed towards the production of a quantum computer. A significant proportion of their recent work has concerned connecting qubits to each other [14, 17]. This naturally requires further experimental advances, for the flux Φ trapped in the loop must be measured (as opposed to the external flux), without introducing an excessive degree of decoherence. Apparently this remains a problem as suggested by the inconclusive results in the latest paper [14].

5 Conclusion

The concept of macroscopic quantum coherence has been reviewed by an investigation into theoretical and even slightly philosophical work, ranging from Schrödinger's famous 'cat paradox' paper to fairly recent ones, most notably those by A. J. Leggett. The central themes in the more recent works have been (a) the theory of distinguishing between coherent superpositions and statistical mixtures and (b) the problems due to decoherence in achieving superposition states in practice.

The relevant experimental work based on this theoretical framework has also been examined. There a superconducting loop interrupted by Josephson junctions is used to create a double potential well of a single macroscopic variable Φ, the magnetic flux trapped by the loop. The two bound states can form coherent superpositions by tunneling between the wells. An energy difference observed between the even and odd superpositions indicates that a statistical mixture is not an adequate model for the system.

It is a general conclusion that, despite some philosophical arguments, a coherent quantum superposition of macroscopic states is a real, observed phenomenon: the two bound states in the superconducting loop are different by a current of order 1 μA, a fairly macroscopic quantity. However, the question is left open whether such effects can be extrapolated into arbitrarily large values. Nevertheless, this is promising news for the development of quantum computers, which by exploiting the parallelism of several quantum states can drastically outperform conventional computers in certain applications.

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